An integrated location-inventory model for the healthcare supply network under stochastic demands

Guoqing Zhang[1]*, Mohammed Almanaseer[1], Xiaoting Shang[1,2]
[1] Supply Chain and Logistics Optimization Research Centre, Department of Mechanical, Automotive & Materials Engineering, University of Windsor, Windsor, Ontario, Canada
[2] School of Traffic and Transportation, Beijing Jiaotong University, Beijing, China
*Corresponding author. Email: gzhang@uwindsor.ca

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Introduction

An efficient supply network in the healthcare system not only offers high contribution to economy, but also plays key roles to promote the health and well-being of citizens. It is hopeful that healthcare system offers the right products to the right patients at the right time. However, due to the complexity of the healthcare system, it will face some significant challenges, such as the timeliness for delivery, accuracy for products, uncertainty for demand, and so on.

To reduce the investment cost and improve the customer satisfaction, vendor managed inventory (VMI) was considered into the healthcare system (Bhakoo et al., 2012), and later received more attention on making healthcare system more demand driven (Krichanchai and MacCarthy, 2017). When the manufacturers and vendors utilize VMI contracts to increase their market share, the uncertainty of demand always affects the healthcare inventory and service level, where VMI contracts become the tools to switch potential lost sales into backorders, thus, improving stock-out management (Yao et al., 2010). Applying the stochastic variables is an important method to describe the uncertainty of demand using VMI policy, and is effective for many research area (Rad et al., 2014; Alawneh and Zhang, 2018).

Motivated by a real-world problem arising from a world-leading medical implants company, we design an integrated location-inventory supply network in a healthcare system under stochastic demands, where VMI policy and direct delivery policy are combined to offer products to hospitals, as is shown in Figure 1. The company supplies three types of medical implants (a heart valve, an artificial knee, and hip) for 147 hospitals located in a province in Canada. The vendor implements a VMI contract for the selected hospitals by integrating the location of the storage facility with replenishment policy, and offers direct delivery for hospitals without warehouses.

The problem statement: Considering the demand uncertainty, the vendor needs to decide which hospitals to establish VMI's warehouses and optimal inventory policy for the warehouses to minimize the total expected cost.

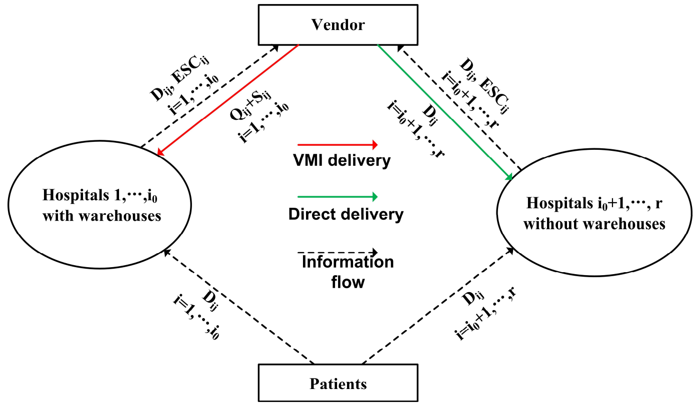

Figure 1. the supply network with VMI delivery and direct delivery

Model formulation

The decision variables include:

Yw_i: 1 if the warehouse set at hospital i, 0 otherwise.

Yd_i: 1 if the direct delivery set at hospital i, 0 otherwise.

Q_{ij}: The quantity of product j delivered to hospital i with assigned warehouse.

SS_j: Safety Stock level of product j at the vendor's warehouse.

S_{ij}: Safety Stock level of product j at the hospital i with assigned warehouse.

R_{ij}: Reorder point level of product j at the hospital i with the assigned warehouse.

SF_{ij}: Safety Stock factor for product j at the hospital i with the assigned warehouse.

m_i: Number of orders delivered to hospital i with the assigned warehouse.

An integrated location-inventory optimization problem under stochastic demand can be modeled as following.

$$Min\ ETC_{VMI} = \sum_{i=1}^{r}\sum_{j=1}^{n} G_{ij}\, D_{ij} Yd_i + \sum_{i=1}^{r}\sum_{j=1}^{n} h_{ij}\left(\frac{Q_{ij}}{2} + (R_{ij} - \mu_{ij})\right) + \sum_{j=1}^{n} h_j\, SS_j + \sum_{i=1}^{r} FC_i Yw_i + \sum_{i=1}^{r} K_i m_i \; + \sum_{i=1}^{r}\sum_{j=1}^{n} \rho_j\, m_i \left(\int_{R_{ij}}^{\infty} f(x_{ij})\,(X - R_{ij})\, d(x_{ij})\right) \quad (1)$$

s.t.

$$Yd_i + Yw_i = 1, \quad \forall i = 1, \cdots, r \quad (2)$$

$$\sum_{j=1}^{n} \left(Q_{ij} + (R_{ij} - \mu_{ij})\right) Vo_j \le Yw_i F_i \qquad \forall\, i = 1, \dots, r \quad (3)$$

$$Q_{ij} \le Yw_i M \quad \forall\, i = 1, \dots, r, j = 1, \dots, n \tag{4}$$

$$SS_{ij} \ge m_i \left(\int_{R_{ij}}^{\infty} f(x_{ij}) \left(X - R_{ij}\right) d(x_{ij}) \right) \quad \forall\, i = 1, \dots, r, j = 1, \dots, n \tag{5}$$

$$SS_j = Z_j \sigma_j \qquad \forall\, j = 1, \dots, n \tag{6}$$

$$\sigma_j = \sqrt{\sum_{i}^{r} (\sigma_{ij})^2\, Yd_i} \qquad \forall\, j = 1, \dots, n \tag{7}$$

$$m_i \le Yw_i M \qquad \forall\, i = 1, \dots, r \tag{8}$$

$$Q_{ij} m_i = D_{ij} Yw_i \qquad \forall\, i = 1, \dots, r, j = 1, \dots, n \tag{9}$$

$$Q_{ij}, m_i,, SS_j, R_{ij} \ge 0 \qquad \forall\, i = 1, \dots, r\,, j = 1, \dots, n \tag{10}$$

$$Yd_i, Yw_i \in [0,1] \qquad \forall\;\, i = 1, \dots, r \tag{11}$$

The objective function (1) is to minimize the total annual expected cost, including the ordering, holding, transportation, setup and shortage costs. Constraints (2) ensure that there is a storage facility or a direct for every hospital. Constraints (3) ensure the warehouse space capacity, and constraints (4)-(5) give the upper bound of order quantity and the lower bound of safety stock level, correspondingly. Constraints (6) calculate the safety inventory, and constraints (7) calculate the standard deviation of demand. Constraints (8) ensure the upper bound of the number of orders, and constraints (9) calculate the demand quantity. Constraints (10)-(11) give the variable domains.

Then, we will discuss the solution approaches for both uniform and normal demand distributions.

Case 1: The demand follows the uniform distribution of (0, D_{ij}), where the lower limit of the uniform demand distribution is zero and the upper limit is D_{ij}.

With case 1, the first model can be transformed as following.

$$Min\, ETC_{VMI} = \sum_{i=1}^{r}\sum_{j=1}^{n} G_{ij}\, D_{ij} Yd_i + \sum_{i=1}^{r}\sum_{j=1}^{n} h_{ij} \left(\frac{Q_{ij}}{2} + SS_{ij}\right) + \sum_{j=1}^{n} h_j\, SS_j + \sum_{i=1}^{r} FC_i Yw_i + \sum_{i=1}^{r} K_i\, m_i + \sum_{i=1}^{r}\sum_{j=1}^{n} \rho_j\, m_i \left(\frac{D_{ij}}{2} - R_{ij} + \frac{{R_{ij}}^2}{2D_{ij}}\right) \tag{12}$$

s.t. (2)-(4), (6)-(11)

$$SS_{ij} \ge m_i \left(\frac{D_{ij}}{2} - R_{ij} + \frac{{R_{ij}}^2}{2D_{ij}}\right) \quad \forall\, i = 1, \dots, r, j = 1, \dots, n \tag{13}$$

$$R_{ij} = \mu_{ij} + SF_{ij} \sigma_{ij} \qquad i = 1, \dots, r, j = 1, \dots, n \tag{14}$$

Where $\int_{R_{ij}}^{\infty} (x_{ij} - R_{ij})\, f(x_{ij})\, d(x_{ij}) = \left(\frac{D_{ij}}{2} - R_{ij} + \frac{{R_{ij}}^2}{2D_{ij}}\right)$.

Case 2: The demand is a normally distributed and the lead time is fixed, where $SS_{ij} = SF_{ij}\sigma_{ij}$, $R_{ij} = \mu_{ij} + SF_{ij}\sigma_{ij}$.

With case 2, the first model can be transformed as following.

$$Min\ ETC_{VMI} =$$

$$\sum_{i=1}^{r}\sum_{j=1}^{n} G_{ij}\, D_{ij}\, Yd_i + \sum_{i=1}^{r} K_i m_i + \sum_{i=1}^{r}\sum_{j=1}^{n} h_{ij}\left(\frac{Q_{ij}}{2} + (SF_{ij}\sigma_{ij})\right) + \sum_{j=1}^{n} h_j\, SS_j + \sum_{i=1}^{r} FC_i Yw_i$$
$$+\sum_{i=1}^{r}\sum_{j=1}^{n} \rho_{ij}\ m_i\left\{\frac{\sigma_{ij}}{2}\left(\sqrt{1+SF_{ij}^2} - SF_{iJ}\right)\right\} \tag{15}$$

s.t. *(2)-(4), (6)-(11)*

$$SS_{ij} \geq m_i\left\{\frac{\sigma_{ij}}{2}\left(\sqrt{1+SF_{ij}^2} - SF_{iJ}\right)\right\} \qquad \forall\, i = 1, \ldots, r\, , j = 1, \ldots, n \tag{16}$$

Results

To illustrate the practical application of the proposed model, we will conduct numerical experiments with the actual data of the vendor, by GAMS 25.1- Baron solver software on an Intel(R) Core (TM) i7-4720HQ CPU@ 2.6 GHz with 8 GB RAM. Table 1 gives the results for different distributions of stochastic demand, showing that the uncertainty of the demand directly impacts the location-inventory assignment model.

Table 1. Results for different distributions of stochastic demand

#	Activity	Normal Distribution (SF_{ij} Base)	Uniform Distribution (R_{ij} Base)	Avg. Uniform Distribution (R_{ij} Base)
1	Expected Total Cost	1,001,754.52	1,432,916.78	1,643,999.90
2	Cost Increase	0.00%	30.09%	39.07%
3	Y_{wi}	87	74	63
4	Y_{di}	60	73	84
5	SS_j	60	81	30
		56	74	29
		54	73	27
6	S_{ij}	329	663	1281
		294	547	1068
		299	587	1151
7	Q_{ij}	2268	2142	2072
		1895	1789	1729
		2030	1920	1862
8	m_i	156	137	115

9	SF_{ij}	1.2167	1.0000	1.0058
		1.3570	1.0000	1.0101
		1.2947	1.0000	1.0081
10	R_{ij}	1787	2068	1961
		1545	1729	1636
		1612	1856	1763
11	ESC_{ij}	54	517	503
		43	431	419
		48	464	451
12	GAMS Solver	BARON	BARON	BARON
13	CPU Time used (s)	6.95	9.481	11.2
14	CPU Time Increase	0.00%	26.70%	37.95%
15	Absolute gap (optca = 1E-9)	625.4539131	891.7635024	1010.472942
16	Relative gap (optcr = 0.1)	0.089319324	0.089672233	0.087301377

Conclusion

Motivated by the actual vendor problem, this research designs an integrated location-inventory healthcare supply network under stochastic demand. Considering VMI policy and direct delivery policy simultaneously, we proposed a mixed integer nonlinear mathematical model, aiming to minimize the expected total cost. To the best of our knowledge, our research is the first effort in modeling and integrating VMI and direct delivery policies in the healthcare network under stochastic demand environment. Subsequently, we discussed the solution approaches for both uniform and normal demand distributions. Lastly, we conducted numerical experiments with the real-world problem, to further illustrate the practical application of the proposed model. Further directions will include solution algorithms for large-scale instances.

References

Alawneh, F., Zhang, G., 2018. Dual-channel warehouse and inventory management with stochastic demand. *Transportation Research Part E: Logistics and Transportation Review*, **112**: 84-106.

Bhakoo, V., Singh, P., Sohal, A., 2012. Collaborative management of inventory in Australian hospital supply chains: practices and issues. *Supply Chain Management: An International Journal*, **17** (2): 217-230.

Krichanchai, S., MacCarthy, B.L., 2017. The adoption of vendor managed inventory for hospital pharmaceutical supply. *The International Journal of Logistics Management*, **28** (3): 755-780.

Rad, R.H., Razmi, J., Sangari, M.S., Ebrahimi, Z.F., 2014. Optimizing an integrated vendor-managed inventory system for a single-vendor two-buyer supply chain with determining weighting factor for vendor′ s ordering cost. *International Journal of Production Economics*, **153**: 295-308.

Yao, Y., Dong, Y., Dresner, M., 2010. Managing supply chain backorders under vendor managed inventory: An incentive approach and empirical analysis. *European Journal of Operational Research*, **203** (2): 350-359.